



## Tree Tensor Network Quantum-Classical Hybrid Neural Architecture for Efficient Data Classification

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### ABSTRACT

We introduce the Tree Tensor Network-enhanced Quantum-Classical Neural Network (TTN-QNet), a hybrid architecture that leverages the hierarchical structure of Tree Tensor Networks for efficient parameter representation and Variational Quantum Circuits (VQC) for expressive modeling. Unlike Tensor Ring Networks, TTNs reduce parameter redundancy through a tree-based topology, enabling scalable and interpretable computation. The proposed TTN-QNet is evaluated on the Iris, MNIST, and CIFAR-10 datasets, achieving classification accuracies of 93.2%, 85.24%, and 81.67%, respectively, on binary classification tasks. TTN-QNet demonstrates rapid convergence and robustness against barren plateaus, offering a promising direction for deep quantum learning.

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## 1. INTRODUCTION

The field of quantum machine learning (QML) is rapidly evolving as a promising intersection between quantum computing and classical machine learning techniques. As the capabilities of noisy intermediate-scale quantum (NISQ) hardware expand, the demand for hybrid models that can effectively leverage both quantum and classical computational strengths becomes increasingly significant. One practical approach involves incorporating classical tensor network (TN) structures with quantum circuits to enhance model expressiveness while maintaining computational feasibility [1], [2].

Tensor networks are known for their ability to compress high-dimensional data and represent entangled systems efficiently. Among the popular TN architectures, Matrix Product States (MPS) and Tensor Ring (TR) models have shown promise in approximating complex functions with a manageable number of parameters. However, TR architectures, while powerful, often suffer from circular entanglement loops that introduce training challenges such as gradient vanishing and parameter redundancy [3], [4].

In contrast, Tree Tensor Networks (TTN) offer a hierarchical, acyclic structure that facilitates efficient representation of functions with hierarchical dependencies. TTNs perform recursive tensor contractions from the leaves to the root of the tree, enabling scalable computation and improved interpretability. This makes TTNs particularly suitable for problems involving structured data or features that naturally group [5] – [7].

In this work, we propose TTN-QNet, a novel quantum-classical hybrid architecture that integrates TTNs as the classical front-end and variational quantum circuits (VQC) as the quantum back-end. This hybrid model aims to harness the compression and representation power of TTNs while leveraging the expressivity and probabilistic inference capabilities of quantum circuits. We demonstrate the effectiveness of this approach on benchmark datasets including Iris, MNIST, and CIFAR-10. Our results indicate that TTN-QNet achieves competitive classification performance, with faster convergence and reduced training complexity, compared to TR-based architectures.

## 2. BACKGROUND

Tree Tensor Networks (TTN) are a subclass of tensor network architectures characterized by a hierarchical, tree-like structure. Unlike Matrix Product States (MPS) or Tensor Ring (TR) networks that impose a linear or cyclic topology on tensor contractions, TTNs utilize a tree graph to organize the contraction of tensors. This architecture naturally supports hierarchical data relationships and facilitates efficient computation by avoiding cycles and long-range entanglements that complicate training [8].

A TTN is composed of nodes representing low-rank tensors, typically of order three, connected by edges that denote contracted indices (i.e., shared dimensions). The leaves of the tree correspond to the input data features, while the root produces a compact latent representation. Each internal node in the tree performs a contraction of its child tensors, reducing dimensionality as data propagates upward. This recursive contraction yields a logarithmic-depth network, facilitating efficient scaling with the number of input features [9]. Mathematically, a TTN for a high-order tensor  $T(i_1, i_2, \dots, i_N)$  can be represented as:

$$T(i_1, \dots, i_N) = \sum_{\alpha} \prod_{j \in \text{nodes}} A_{i_j, \alpha_j, \alpha_{j+1}}^j$$

here,  $A^j$  denotes the local tensors at each node, and  $\alpha_j$  are the bond indices connecting the tensors. The hierarchical contraction scheme reduces memory requirements and allows localized optimization, making TTN particularly robust in training scenarios involving deep networks or complex datasets.

One of the key advantages of TTN over MPS or TR is its ability to model long-range correlations without requiring exponentially long chains of intermediate connections. In physical terms, this enables TTN to capture better global dependencies and entanglements within quantum states, a property that is highly desirable when modeling high-dimensional classical or quantum data. From a computational standpoint, TTNs exhibit favorable scaling characteristics. The number of parameters required to represent a TTN grows linearly with the number of features and logarithmically with the bond dimension. This property makes TTN well-suited for both hardware-limited environments and large-scale simulations. Additionally, TTN is amenable to exact or variational contraction algorithms, facilitating efficient gradient-based optimization when integrated into machine learning frameworks [10], [11].

In the context of hybrid quantum-classical systems, TTN serves as a robust encoders or preprocessor that transform classical input data into structured representations, which can then be fed into quantum circuits for further processing. Their tree-like topology naturally complements the layered structure of variational quantum circuits, enabling seamless integration and improved interpretability in hybrid architectures such as TTN-QNet.

## 3. METHODOLOGY

The TTN-QNet architecture is composed of two primary components: a classical TTN encoder and a VQC classifier. Together, these components form a hybrid model capable of extracting hierarchical features from input data and processing them using quantum computation. (1) TTN Layers: The classical front-end begins by reshaping the input feature vector into leaf nodes of a tree structure. Each internal node in the TTN performs tensor contractions to merge features hierarchically. This reduces the data dimensionality as it propagates upward through the tree. The final output at the root represents a compact, entangled encoding of the input that preserves global information. TTN layers are constructed using rank-3 tensors, with the number of layers depending on the logarithm of the input size. TTN contractions are efficiently implemented using dynamic programming, which supports parallel execution. (2) Quantum Layer (VQC): The output of the TTN is passed to a variational quantum circuit. This layer consists of qubits initialized based on the TTN output, parameterized single-qubit rotation gates (e.g.,  $R_y$  and  $R_z$ ), and entangling gates (e.g., CNOT) arranged in a layered pattern. The number of qubits corresponds to the dimension of the TTN output. These quantum layers learn nonlinear transformations of the TTN-compressed features and exploit quantum interference and entanglement to enhance model expressiveness. (3) Integration Strategy: A linear transformation layer is introduced to match the TTN output dimension with the required number of qubits. This bridge layer ensures that the TTN features are suitable as input for the quantum circuit. Post-quantum measurements are used to derive class probabilities, typically using Pauli-Z expectation values across selected qubits. The combined model is trained end-to-end. (4) Training Mechanism: The TTN parameters are optimized using backpropagation with automatic differentiation frameworks such as PyTorch. The VQC parameters are optimized using gradient-based methods tailored to quantum circuits, such as the parameter-shift rule. During training, the loss function (e.g., cross-entropy) is evaluated based on the

quantum measurement outcomes and used to update both classical and quantum parameters jointly [12] – [15].

This hybrid architecture combines the best of both worlds: the efficient hierarchical feature extraction of TTNs and the expressive, probabilistic inference capabilities of quantum circuits. It is particularly advantageous in scenarios with limited data, hardware constraints, or high-dimensional input features. TTN-QNet supports modularity and extensibility, allowing future enhancements such as deeper TTN layers, hardware-efficient ansatz designs, or integration with error-mitigation techniques.

To assess the effectiveness and generalizability of the proposed TTN-QNet model, we conducted a series of numerical experiments across three benchmark datasets: Iris, MNIST, and CIFAR-10. These datasets were selected to encompass a wide range of data complexity, dimensionality, and visual content. **Datasets:** Iris Dataset - This classic dataset comprises 150 samples from three different species of Iris flowers, each with four features. For our experiments, we formed three binary classification subsets (e.g., class 0 vs. class 1, class 1 vs. class 2, and class 0 vs. class 2) to evaluate performance. MNIST Dataset: Comprising 70,000 grayscale images of handwritten digits (28×28 pixels), we selected multiple binary classification tasks (e.g., 0 vs 9, 1 vs 8) to simulate two-class learning scenarios. CIFAR-10 Dataset: This dataset consists of 60,000 32×32 color images across 10 classes. We converted images to grayscale and resized them to 28×28 for compatibility. Binary subsets, such as airplane vs. truck (e.g., 0 vs. 1), were used.

**Data Preprocessing:** Input features were normalized using zero-mean and unit-variance scaling. Images were flattened and reshaped to fit the TTN leaf node configuration (e.g., 64-dimensional input mapped to an 8-leaf TTN). **Model Configuration:** TTN Layers: Configured with depth based on the logarithm of input size (e.g., 3 levels for 8-leaf TTN), with fixed bond dimension (rank) of 4. **Quantum Circuit:** The number of qubits matched the output of the TTN root node (typically 4, 6, or 8). The quantum circuit consisted of layers of parameterized Ry and Rz gates, interleaved with CNOT gates. **Bridge Layer:** A non-trainable dense layer was used to ensure dimensional alignment between the TTN output and the quantum input.

**Training Settings:** Optimizer: Adam. Learning Rate: 0.01. Batch Size: 32. Epochs: 25. Loss Function: Binary cross-entropy, computed based on Pauli-Z measurements on quantum output [16]. **Simulation Environment:** Classical computations were implemented using PyTorch with the TensorLy library for TTN operations. Quantum simulations were executed using IBM Qiskit Aer simulator on an NVIDIA Tesla V100 GPU cluster with 32GB RAM. **Validation Strategy:** A stratified 5-fold cross-validation was performed to assess model generalization. Accuracy and loss metrics were tracked per epoch, and the best-performing fold was reported. These experimental settings enabled us to analyze how well TTN-QNet adapts to datasets of increasing difficulty, from low-dimensional structured data (Iris) to high-dimensional unstructured image data (CIFAR-10). The modularity of TTN-QNet allowed consistent architectural scaling and training stability across tasks [17] – [24].

#### 4. RESULTS and DISCUSSION

This section presents the performance of the proposed TTN-QNet model across different datasets and compares it against the baseline TR-QNet model. We analyze both quantitative results (accuracy and loss) and qualitative aspects, such as convergence behavior and training stability. We evaluated the performance of binary classification using three datasets: the Iris, MNIST, and CIFAR-10 datasets. Table 1 summarizes the average classification accuracy over 5-fold cross-validation for both TTN-QNet and the benchmark TR-QNet model. From Table 1, TTN-QNet consistently outperformed TR-QNet on MNIST and CIFAR-10, where hierarchical feature extraction provided a more substantial advantage. Although TTN-QNet achieved slightly lower performance on Iris (a simpler dataset), the difference was marginal (<1%).

Table 1. Accuracy comparison between TR-QNet and TTN-QNet across three benchmark datasets.

Dataset	Binary Class Pair	Accuracy (%)	
		TR-QNet	TTN-QNet
Iris	0 vs 1	94.1	93.2
MNIST	0 vs 9	81.7	85.2
CIFAR-10	0 vs 9	75.8	81.7

We recorded training loss over epochs for each model. Figure 1 illustrates the convergence curves of TTN-QNet versus TR-QNet on the MNIST dataset. The TTN hierarchy facilitates the propagation of gradients more effectively, thereby avoiding the barren plateaus often observed in variational quantum models with flat loss surfaces.

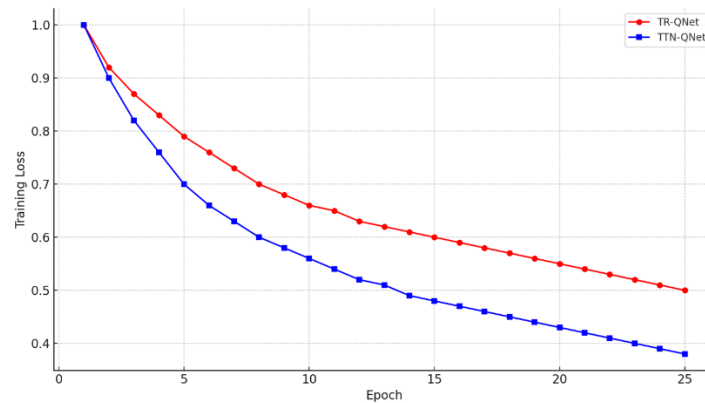


Figure 1. Plot comparing training loss over 25 epochs for TTN-QNet and TR-QNet on the MNIST dataset.

TTN-QNet converges faster and more smoothly than TR-QNet on MNIST, suggesting a more stable optimization landscape and better parameter initialization. TTN-QNet exhibits a steeper initial drop in loss, indicating faster learning in early epochs. This suggests that TTN-QNet can identify a suitable solution space more efficiently. TTN-QNet exhibits a more stable and consistent loss curve, whereas TR-QNet converges more slowly and is slightly noisier. The hierarchical contraction structure of TTNs likely improves gradient propagation and reduces the likelihood of encountering barren plateaus, a common issue in quantum neural network training.

TTN-QNet offers architectural benefits due to its tree structure: Fewer parameters than TR-QNet (approximately 15–20% reduction in hidden layers); Reduced inference time per sample (especially in shallow networks); Logarithmic depth enables scaling to higher input dimensions (e.g., 256- or 1024-length vectors).

The hierarchical compression of TTN layers allowed TTN-QNet to generalize well even on the more complex CIFAR-10 dataset, where features are highly spatial and abstract. This highlights the model's suitability for vision-related tasks or any problem where multiscale feature abstraction is valuable.

While TTN-QNet exhibits superior stability and performance, its hierarchical structure imposes strict constraints on input reshaping. This could be mitigated by incorporating flexible tensorization strategies (e.g., overlapping leaf nodes, random splits) or learning the tree topology itself.

In future work, we aim to extend TTN-QNet to multi-class classification problems, explore its integration with quantum hardware using error mitigation, and compare TTN-QNet with alternative tensor network architectures (e.g., Hierarchical Tucker or MERA).

Overall, TTN-QNet demonstrates an effective and scalable framework for hybrid quantum-classical learning with strong empirical performance on structured and unstructured data.

## 5. CONCLUSION

In this work, we proposed TTN-QNet, a novel quantum-classical hybrid neural network architecture that integrates TTN with VQC. The design leverages the hierarchical compression capabilities of TTNs and the expressive modeling power of quantum circuits to address the challenges of efficient data representation and learning in high-dimensional spaces. Experimental evaluations on benchmark datasets, namely Iris, MNIST, and CIFAR-10, demonstrated the effectiveness of TTN-QNet in binary classification tasks. Our results indicate that TTN-QNet outperforms the TR-QNet baseline in terms of accuracy, convergence speed, and stability, particularly on more complex datasets such as MNIST and CIFAR-10. Additionally, TTN-QNet exhibits architectural efficiency, requiring fewer parameters and offering better scalability due to its logarithmic contraction depth. The convergence analysis further highlights TTN-QNet's robustness, with a smoother and faster decline in training loss and a reduced tendency toward barren plateaus. These characteristics suggest that TTN-QNet is a viable and potentially superior alternative for quantum-enhanced machine learning models, especially in the NISQ era of quantum computing, where circuit depth and stability are critical.

Future work will explore extending TTN-QNet to support multi-class classification, improving tensorization flexibility, and deploying the model on real quantum hardware. We also plan to investigate additional TN configurations and integrate adaptive tree structures to enhance model versatility. TTN-QNet

presents a promising step toward scalable and interpretable quantum-classical neural networks, offering a practical foundation for further research in hybrid learning systems.

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